Analysis of Flexible Diaphragms

Introduction

The general principle underpinning the entire diaphragm analysis process is simple statics. You can break the diaphragm up into smaller elements bounded by collectors/struts and chords and resolve the forces across each small two dimensional rectangle. This gets you the design planar shear forces and allows you to calculate axial chord/collector and shear wall loads. The complicated part is ensuring that every element is determinant, either with boundary conditions or adjacent elements. The following steps will lay out boundary conditions and walk through element by element while explicitly discussing this general principle, since it is more useful to describe the specifics of each situation as it arises. It is important however, to keep simple statics in mind, if you are ever confused make sure your free body diagrams are clear and remember your equilibrium equations.

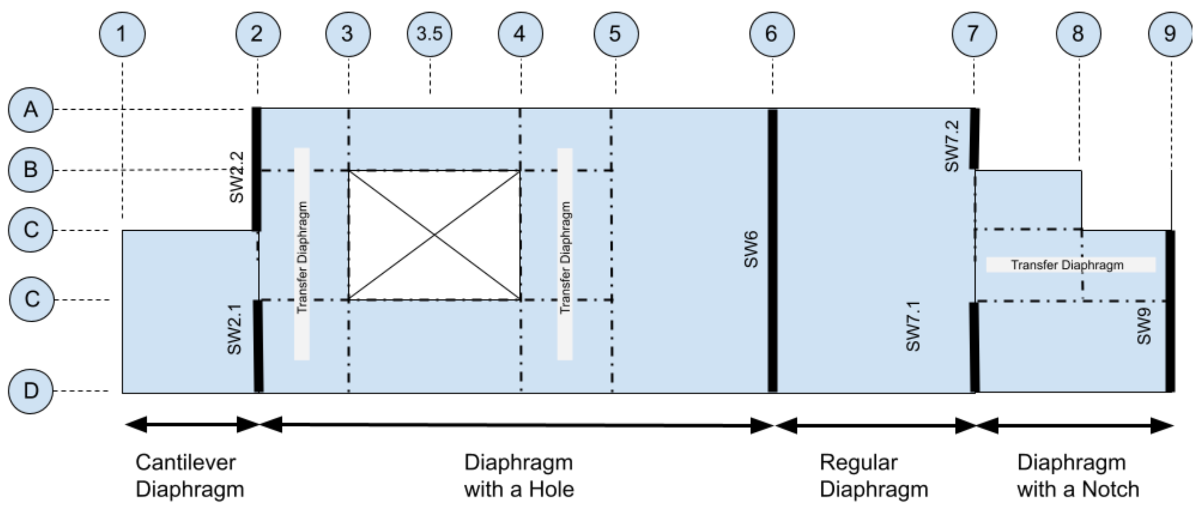
At the end of this process you should refer to your work as an example; there are a lot of steps and complicated parts but once you have a direction working though it should be easy. This is especially important since you'll be doing it a lot. For a diaphragm with a hole there are *six* loading cases, two dimensions each with seismic, wind one way, and wind the other way. For regular diaphragms and notches there are only two, since wind direction doesn't matter you can pick the largest of wind or seismic. You can't do this simplification for a hole since seismic might apply a larger load to the whole diaphragm, but wind in one direction might apply a larger load to a part of the hole, so you have to check. Of course this is also all based on the assumption that all the important members fall on a grid. What this means is that it is important to use your tools to the best of your ability, automate and simplify what you can. This writeup goes through all the steps by hand, from given seismic and wind loads to all your design diaphragm capacities, but this is mostly for understanding the process; cut out as much work as you can once you understand it.

1 Problem Setup

All diaphragm problems begin with the same setup steps. These are important for both ensuring consistency and accuracy in results, as well as cutting out repetitive work. No matter how many diaphragms a structure has or how many complexities each diaphragm has you will always refer back to the shear diagram.

1.1 Design the Diaphragms

This is where the book, *The Analysis of Irregular Shaped Structures Diaphragms and Shear Walls*, comes in handy, specifically the first two chapters. The design requires understanding the limitations of a diaphragm, namely that it has little tension/compression capacity but lots of shear capacity. Doing the design before the calculations seems like a risk, since if the capacities don't meet the demands we will need to repeat the whole process (and that could happen), but the design code aspect ratio requirements and nature of your structure will mean it is likely that your initial design without calculation is sufficient and the calculations rigorously confirm this. This is similar to how we size a beam with assumed dead loads, then want to verify the beam will still work with its exact self weight. There are also details like the length of a collector or the width of a transfer diaphragm that can be altered in reaction to the calculations. Furthermore if you have designed the diaphragm reasonably and the demands still exceed the capacities, look at increasing the capacities with nailing patterns, blocking, stronger collectors, etc.



This diagram shows a designed floor diaphragm for one direction of loading (top to bottom), the shear walls and distributed load for the other direction have been omitted. For the purposes of this example a wind load was chosen, but do keep in mind that wind or seismic forces could control. With simple or notched diaphragms the greater force controls and the choice is simple, for diaphragms with holes this is not the case. The load is distributed differently to the different parts of the hole in wind versus seismic loads so each case must be checked regardless of if one is larger than the other.

Write out a consolidated collection of design aspect ratios here, maybe some other design restrictions and things too.

Shear walls cant be less than height/3.5

Hole(maybe notch too) transfer diaphragms cant be more than 3:1, or 4:1 with blocking

Holes aspect ratios above and below the holes

Hole max sizes?

Cantilevers cant be more than 25ft

Cantilevers max 0.67:1 aspect ratio for two stories

Cantilevers max 1:1 aspect ratio for one story without deflection calculations

Cantilevers max 1.5:1 aspect ratio for one story with deflection calculations and diagonals

1.2 Make a Shear Diagram (Ignoring Holes/Notches)

You only need a shear diagram for the analysis diaphragm but it's easy enough to make it for the whole building and will be useful later. We're going to define a single diaphragm as one surface between two supporting shear walls in the analysis direction of loading (yes, we will need to do this for each direction). This means you will likely have completely different diaphragms in each direction. This surface can have holes in it or notches in the corners, just not another parallel shear wall, in that case you'd have two separate diaphragms. For holes and notches we assume the diaphragm continues and forms a complete solid rectangle. If there is only one shear wall and the diaphragm cantilevers don't divide the shear by 2 in the following equation. When you have multiple discontinuous shear walls in the same line we will just be considering the whole line as a shear wall for now and distribute the forces appropriately later. It is important to nail down consistent length measurements in both dimensions for all your diaphragms, holes/notches, and spans from shear walls to holes/notches. Make sure your hole/notch and span measurements add up to your total diaphragm measurements. This is a quick and easy way to minimize your total error. Put yourself in a position where you don't have to reference the model, you don't have to worry about inside/outside/on center measurements, and where you don't have to worry about your sub measurements adding up to the total measurements. It would also help to use consistent labels for gridlines, taking note of the gridlines on the original drawings or assigning your own. This allows for the use of easy to read variables, properly describing your equations, and more easily showing your work. *It is absolutely critical that you set yourself up for success with a good problem setup,* the number of steps are plentiful and the room for error is significant.

Find Shears

Diaphragms can be thought of as really thick simply supported beams, so the simple beam equations often work the same. The key equation for designing the shear diagram is:

$$V = \frac{w * L}{2}$$

Where V (lbs) is the shear on one side of a wall, w (plf) is the applied load on the diaphragm and L (ft) is the length between the two shear walls. Remember, when taking L we assumed the diaphragm is a rectangle. Note that the shear is equal and opposite at each edge of our rectangular diaphragms (as long as w is constant), so you only need to do half the calculations. As already stated, don't divide a cantilever's shear by 2 since its tributary area is the whole diaphragm not half. A more accurate equation would be: $V = w * L_t$ where L_t is the tributary length of the shear wall into the analysis diaphragm, but we are usually given lengths between walls so using the earlier equation is often simpler than finding and keeping track of all the tributary lengths.

When you have a change in w, like a notch for seismic loads or the corner effects for wind loads, the solution is still quite simple. This changes the slope of the shear graph and you can simply solve the shear at that point with the first equation up to the point of change, then starting from that shear, continue with the new steeper slope to the end. We are simply making a shear graph where V is the y axis and w is the slope over any section.

Note that shear walls in the middle of the structure can have two different tributary lengths or loads, and therefore two different shears for the diaphragms on each side. Not only that but the sign of the graph will likely flip as you cross from the right edge of a diaphragm to the left edge of a new diaphragm. This represents the discontinuity in the shear graph at a support, meaning that the supporting force of the shear wall is the difference between the shear on the left and the shear on the right (this is a useful value we'll come back to later). A shear wall cannot have different tributary lengths along the *same* side, again holes/notches are treated as continuous rectangles. In regards to signs, we walk along a shear graph left to right, accumulating shear force in the direction of the load, and then jumping in the direction opposite the load at a point of support. What this means is that usually (though not always if you have a very negative or very positive section of the graph) you will have a positive shear on the left side of the analysis diaphragm and a negative shear on the right side, and a negative applied force (the direction doesn't matter since lateral forces are reversible, it's just easy to compare to 'downwards' gravity loads), with a negative linear slope down to the right, joining those two points. That being said, don't draw the graph yet as there are more steps.

Also note that we do not need to worry about adjacent bending moments turning any support shears positive, as each diaphragm span is analyzed as a separate simply supported beam with zero moment across the supports. For cantilevered sections we must have a moment at the support, but we just provide it all from the support and perpendicular shear walls and we don't transmit it across to adjacent diaphragms. This is a simplification that is under active debate, since in reality the separate diaphragm spans do act as one continuous beam, but we'll still simplify it since it is still considered acceptable.

Find Shears per Unit Length

Once we have the shears at each side of the shear walls we want to convert those to shears per unit length of the wall in order to account for a diaphragm on one side of the wall being a different depth into the structure than the diaphragm on the other side of the wall. For this we use:

$$v = \frac{V}{d}$$

Where v (plf) is the shear per unit length, V (lbs) is the previously calculated shear, and d (ft) is the depth of the diaphragm along the shear wall. Depth is the length of the diaphragm perpendicular to the tributary length. This is the actual length of the diaphragm along the shear wall not assuming it's rectangular; if a notch cuts out a corner and the length of one shear wall is shorter than the other, then the diaphragm is acting across a shorter depth (but it's still applying the greater weight from the shear diagram as if the notch wasn't there). If you have a change in tributary length from one side of the shear wall to the other, and a change in depth from one side to the other, you should end up with different V's and v's on each side of the wall.

When there is a hole or a notch, we need to also compensate for the change in depth at the edge(s) not touching a shear wall. Note that V will be the same on each side of the collector since there is no support reaction, but v will change as the depth has changed. First we want to find what the shear is at each of the collector(s) of the hole/notch, one collector at the edge of a corner notch, two for each edge of a hole or intermediate notch. Recalling the basics of a shear graph we can start at a shear wall on the side containing our hole/notch near the analysis collector and walk along the linear shear graph using the constant load w as the slope.

$$V_{collector} = V_{support} + w * l_{to\ collector}$$

Were $V_{collector}$ (lbs) is the shear at the line running along the collector of the hole/notch, $V_{support}$ (lbs) is the shear we already found at a shear wall on the side which we are analyzing, w (plf) is the applied load, and $l_{to\ collector}$ (ft) is the distance between the support and the collector. $l_{to\ collector}$ assumes that there are no other complexities between the support and the collector. If there are, you need to solve that complexity first on the shear diagram and then treat that as $V_{support}$. Note that we need to consider signs here, the plus assumes that we are walking left to right along the shear graph. If we have a positive shear on the left support and a negative distributed force, we will be subtracting from the shear and walking *down* the graph. The shear found at the collectors should always be somewhere between the shear of the two supports. Once you have the shears at the collectors we want to find the shear per unit length at each side of each collector. We use the same equation as before, $v = \frac{V}{d}$, where the main difference in depth we're considering is the loss/gain in depth due to the hole/notch. For a hole we do still need to consider the depth both above and below the hole. The shear per unit length on the side with the hole/notch should be greater than on the continuous side, as there is less length for the shear to distribute over. One way to think about this is that the same amount of shear (calculated assuming it's rectangular) is compressed into a narrower area increasing the "density" of the shear. The shear per unit length graph over a hole should return back to where the shear per unit length graph of the main diaphragm would be without the hole, and the slopes over the hole versus the full diaphragm should be different. The change in shear per unit length for a notch should be compensated for by the shorter depth of the diaphragm itself, ensuring the shear diagram still all connects. Again note that shear (V) will be the same across collectors, only shear per unit length (v) will change. If the v graph is not intuitive remember that the slope of the graph depends on the depth of the diaphragm. The V graph should be intuitive though its less useful for the following analysis.

Graph

Now make the basic shear diagram, the applied load (w) is the linear slope and the solved shears per unit length (v) are the edges of the discontinuities. The system is static, so the graph should close out to zero over each diaphragm and the whole structure, with a margin for rounding error. Each support should have two v's, so when you check to ensure a diaphragm closes to zero you just use its edges v's.

NEED A DIAGRAM HERE

Now that you have the problem setup done you will work through solving each diaphragm based on its type. Find the "Analysis of" section pertaining to one of your given diaphragms, work through it, then move on to the next diaphragm you have and work through its section. Again, if you have a more complex diaphragm or are unsure how to apply the information in the following sections, you can always return to breaking the diaphragm up into elements and solving with basic statics. Once all of the diaphragms are solved (design shears, chord forces, non shear wall axial collector forces), move on to the final work section.

2 Analysis of Simple Diaphragms

With the shear diagram the majority of the work for simple diaphragms is already done. The shear at the edges of the analysis diaphragm from the basic shear diagram is the design shear for the surface, all that needs to be done in this step is calculate chord forces.

2.1 Solve Chord Forces

Again, diaphragms are essentially really thick simply supported beams, and as such we can use similar equations:

$$F = \frac{w * L^2}{8 * d}$$

Where F (lbs) is the maximum force in the chords, w (plf) is the applied load, L (ft) is the length of the span between the shear walls, and d (ft) is the complete depth of the diaphragm parallel to the shear walls. This calculates the maximum axial force due to a bending moment, which is our chord forces. For a simple diaphragm the beam is symmetrical, meaning the force is the same on the top and bottom.

For each 'Analysis of' sections we will solve the design shear forces, the chord forces, and the axial *non-shear wall* collector forces. The shear wall collector forces will be calculated in the final step after all the design shears are found since you need the shears of each adjacent diaphragm of an interior shear wall before you can do that calculation.

3 Analysis of Cantilever or Open Front Diaphragms

Cantilever diaphragms are relatively simple, there's no need for transfer diaphragms or breaking the problem down into smaller elements, *we already have the design shear off of the basic shear diagram*. However, we do need to consider the torsion of the diaphragm and the load that is applied to the perpendicular shear wall lines along its chords (technically struts since there's no bending restraint and therefore no bending "chord" forces). An especially important consideration of cantilever diaphragms is deflection, as cantilevers have significantly more deflection than simply supported beams and the open front nature of the section means that deflection might have significant impact on the gravity systems. Deflections are outside the scope of this writeup, but do look into *The Analysis of Irregular Shaped Structures Diaphragms and Shear Walls* chapter 6 for more information.

3.1 Determine Torsion and Plot Strut forces

The first step is to find the shear force generated by the eccentricity of the applied load and reaction force.

$$V_{strut} = \frac{V_{SW} * L}{2 * d}$$

Where V_{strut} (lbs) is the shear due to strut forces where the strut meets the shear wall, V_{SW} (lbs) is the basic shear diagram shear force at the shear wall on the cantilever side, L (ft) is the length of the cantilever, and d (ft) is the depth of the cantilever along the shear wall. If there are no perpendicular shear walls in the cantilever then this is your design axial strut force. If there are shear walls in the cantilever then the peak load will be reduced and you need to distribute the shear force along the strut.

Distribute the Shear Force

We can use a similar strategy for finding the axial forces along a shear wall in the parallel direction as described in the final work section. We could construct a full net force diagram, but since there's only one side we will go through the steps directly. First we convert V to v_{load} and v_{sw} by dividing V_{strut} by the length of the whole strut and the length of just the shear walls in the strut respectively, $v = \frac{V}{d}$. Next we solve $v_{resistance} =- \left|v_{SW} - v_{strut}\right|$ where $v_{resistance}$ is the slope of the axial force diagram over the shear walls, and v_{load} is the slope of the axial force diagram over the cantilevered struts. The signs break down a little here, theoretically you could use the signs to help tell what's in tension and compression, but we already know the answer to that with basic assumptions about a cantilever so we can just take the v_{load} as a positive slope and the $v_{resistance}$ as a negative slope. The axial force diagram should be zero at the end and some value at the support. This support axial force is the bending moment that the support reaction provides in the form of a force couple. Remember this is not transmitted to adjacent diaphragms under our assumptions. In the case where you do have two adjacent cantilevers you can solve for the actual moment about the support due to the two cantilevers, then convert that moment into a force couple. When you draw the axial force diagram it should be continuous, in other word the axial strut force at the shear wall due to one cantilever should be equal to the axial force at the shear wall due to the other cantilever. Note that the signs will be opposite for the shear on either side of the wall.

4 Analysis of Diaphragms With Corner or Intermediate Notches

Notches are much more complex since there are many different cases. For this reason a general solution that works well for all notch cases is the best way to communicate this information and if you are seeking more specifics go reference chapters 3 and 4 of *The Analysis of Irregular Shaped Structures Diaphragms and Shear Walls*. To outline the steps for a notch problem we first need to draw shear diagrams. If you have a simple area of the diaphragm resting above or below a transfer diaphragm then you will need to break it out into *its own separate* shear diagram. If we treat transfer diaphragms as simply supported beams, then we want to know what the load of this smaller simple diaphragm area is on the beam. Once we draw a basic shear diagram for just this smaller area we can use the discontinuity at the end to treat the area as a point load on the transfer diaphragm. This is also the strut/collector axial force extending into the transfer diaphragm. Once we have this simplification we can resolve the shears across the transfer diagram with statics principles, find the net shears off of the new basic shear diagram, then solve all the axial forces with the same formula.

4.1 Draw New Shear Diagrams and Find Boundary Condition

For notches where the corner strut is perpendicular to the applied load you can use the basic shear diagram. For notches where the corner strut is parallel with the applied load you need to break out the two diaphragms separated by the transfer diaphragm and find their shears separately. As already mentioned when a free area of the diaphragm falls onto the transfer diaphragm beam we need to convert that to a point load at the corner strut/collector.

First we want to find the applied loads on each diaphragm proportional to their depth. Simply multiply the total applied load by the ratio of the depth of the section over the total depth of the diaphragm. The boundary between the two areas is the line running along the corner of the notch and the transfer diaphragm. You will end up with two smaller loads that should add up to the original total load. Now look at the smaller diaphragm bounded by the edge of the transfer diaphragm and the small part of the notch. One side has a shear wall support, the other side has the corner strut. Solve the basic shear diagram the same way as before, with $V = \frac{w*L}{2}$ and $v = \frac{V}{d}$ using the new small applied load, the length between the strut and the collector, and the depth between the load and the edge of the transfer diaphragm.

The larger of the two diaphragms includes the simple diaphragm area and the transfer diaphragm itself. The important part to note here is that the slope changes, the section under the notch still experiences the full applied load, but the section under the smaller diaphragm experiences its proportion of the load. Since we technically don't know the shear wall reaction forces we can’t just walk along the graph, we have to use statics. The sum of moments about one shear wall will give us the force along the opposite shear wall. The specific equation is:

$$\Sigma M_A = 0 \Rightarrow V_C = \left(w_A * l_A * (l_A / 2) + w_C * l_C * (l_C + l_C / 2)\right) * \left(\frac{1}{L}\right)$$

Where w_A is the load over the section A, l_A is the distance to the corner strut from A, and L is the length of the sub-diaphragm we are drawing the shear diagram for (right now we're just analyzing this smaller part as a rectangle). That being said it is most useful to just think about the sum of moments around point A equalling zero and solving for the shear at point C keeping in mind the two distributed loads on the rectangle. With the end reactions we can now walk along from either shear wall with $V_{end} = V_{start} + w * x$ to get V_B, the shear force at the corner collector, making sure to use the correct w depending on which side you start from. Note that signs can be confusing here; the equation for the shears will likely output two positive numbers, but one should be negative. Remember we are drawing a shear graph where the applied load is the slope and the point shears we're solving are the discontinuities in the opposite direction to the load. You must keep this in mind in order to determine the sign of B since it depends on where you start from. Once you have all the points connecting the three dots yields the shear diagram.

Find Boundary Condition With Corner Strut

The strategy here depends on if the corner strut is parallel or perpendicular to the applied load, do we have a shear/axial force or a chord force. When the corner strut is parallel to the force (it runs in the same direction as the force arrows), then your work was already done in step 4.1. The end of the shear diagram of the smaller diaphragm *is* the axial force, being maximum at the actual corner of the notch and closing to zero at the beginning and end of the strut/collector. Tension or compression should be intuitive with the applied load direction and the transfer diaphragm applying resistance.

When the corner strut is perpendicular to the force we solve it as a chord force with the following equation:

$$F = \left(V * x - \frac{w * x^2}{2}\right) / d$$

Where F (lbs) is the maximum force in the corner strut/collector at the corner, V (lbs) is the support shear provided by the shear wall line, x (ft) is the distance from this support to the corner of the notch, w (plf) is the distributed load (of which there's only one in this case), and d (ft) is the depth of the diaphragm including the notch (the shorter depth). You could also just look at the area under the basic shear diagram from the shear wall to the corner, you should get the same result. Signs are less intuitive in this case, ….. Since we can solve for the corner strut/collector force directly we do not need any other boundary conditions and we do not need any other sub-diaphragm, we can move straight on to resolving the transfer diaphragm.

I will now discuss some of the specific notch cases, if this is unnecessary you can skip to 4.2. It is important to lay this out now before further analysis since the cases are all defined in the shear diagrams themselves.

Multiple Notches

All of the following information is written as if there is only one notch in the analysis diaphragm. The method for analyzing a diaphragm with multiple notches is the same as with any other collection of diaphragm complexities. The analysis is entirely based on the shear diagram numbers, so make sure the shear diagram is consistent. You can walk through all the same steps in the following analysis for each notch separately since their interaction is accounted for in the shear diagram numbers. For loading perpendicular to the struts just following the same basic shear diagram. For loading parallel to two struts you'll now need three shear diagrams, proportionally splitting the load two and three ways for each stack of diaphragms. If you have one parallel strut and one perpendicular strut then you analyze the perpendicular strut with the basic shear diagram (that accounts for the other notch already) and you solve the parallel strut with the two smaller shear diagrams.

Intermediate Notches

The analysis is nearly identical to a corner notch. When there is an intermediate notch you will have two transfer diaphragms and two 'corners'. With a parallel strut you will have three small shear diagrams, one for the top section, one next to the notch, and one for the bottom section. This is accompanied by three loads for each section proportional to their depths. In this case both corner strut forces will be equal, and the same equation can be applied using the length and depth of the middle section next to the notch between the transfer diaphragms. This is the section that 'hangs' and applies loads to the transfer diaphragms. With this information you can solve each transfer diaphragm separately in the following steps.

Propped Cantilever Notches or Offset Shear Walls

These are corner or intermediate notches where the shear walls are at the far side wall and at the corner of the notch itself. The notch can then either have a third shear wall at its edge, or it could cantilever out. If the strut/collector at the corner runs through the entire length of the diaphragm, and there's no necessary transfer diaphragm beam in this direction, then the shear wall line splits it into two diaphragms that can be analyzed as separate simple diaphragms or a simple and a cantilever. Loading perpendicular to the corner strut in this case is meaningless since that would fall along a full length shear wall along one of the complete lines of the transfer diaphragm, but loading parallel to a strut could result in the need of a transfer diaphragm if the strut/collector doesn't run full length. These cases require some more thought when making the smaller shear diagrams. The smaller section above the transfer diaphragm is still analyzed the same way, as a simple span with its proportion of the load, but the other diaphragm (including the whole transfer diaphragm) is more complex. When you solve for the reaction forces either you do it as a propped cantilever or two simple spans, then you use that to plot the shear diagram. For two simple spans you can find the reactions with the same tributary method as always, just make sure to use the right w's.

4.2 Solve Transfer Diaphragm

Now that we have the applied load on the transfer diaphragm, we want to figure out what the reaction forces are. Here we can use the sum of moments about one of the ends to get the reaction force at the opposite end:

$$\Sigma M_C = 0 \Rightarrow V_A = \left(F_B * x_C\right) / d$$

Where V_A (lbs) is the shear force at one of the ends of the transfer diaphragm, F_B is the corner strut force calculated in the previous step, x_C (ft) is the distance from V_C to the corner, and d (ft) is the depth of the transfer diaphragm (the shorter depth). We can then use the same equation to get the shear at C, $\Sigma M_A = 0 \Rightarrow V_C = \left(F_B * x_A\right) / d$. Once we have all of the shears we can convert them to shears per unit length with $v = \frac{V}{d}$ where in this case the depth the shear is applied over is the length of the transfer diaphragm, essentially the length that the corner collector extends into the transfer diaphragm. For signs, think about the transfer diaphragm as a simply supported beam with the point load from the corner strut; both reactions are going to oppose the applied load. If the applied load is 'down' and moving left to right, the shear diagram will jump up at the first support, jump down across zero at the load, then jump back up and close out to zero at the final support, meaning you have a positive shear across the left section and a negative shear across the right section.

Find Net Shears

The principle of finding the net shears is fairly simple, adding the applied shears we found in the last step to the basic shear diagram, but it's a bit more complex since it depends on the direction of loading. If the corner strut is perpendicular to the applied load we use the original singular basic shear diagram. Assuming we are loading 'downwards', over each individual smaller element of the diaphragm the shears on the top edge should match the shears on the bottom edge, but the shears on the left edge should not match the shears on the right edge. In other words the shear only changes in the vertical direction at a boundary, but in the horizontal direction both at boundaries and over distance due to the force. This is useful as we only need to note three shears for the transfer diaphragm since the shears on top or on bottom of the corner collector area are already defined by the adjacent elements. The method is still the same though, add the shear in each section of the transfer diaphragm to the basic shear diagram shear to get the net shear.

For a corner strut perpendicular to the applied load the net shears are found the same way, but the pattern is different. Assuming again we are loading 'downwards' (as if we've rotated the whole diaphragm 90 degrees) the shear forces should again only change at boundaries in the (new) vertical direction and change at boundaries and over distance in the horizontal direction. This means that you need four shears over the transfer diaphragm, as the corner collector is a boundary and will have different values at each edge left versus right.

4.3 Solve and Plot Axial Collector and Chord Forces

Usually there are some simplifications for solving one or the other, but since notches are so variable we will use the general form to go from design shear forces to axial boundary forces:

$$F_{axial@key\ point} = F_{previous\ key\ point} + \frac{\left(v_{top\ left} - v_{bottom\ left}\right) + \left(v_{top\ right} - v_{bottom\ right}\right)}{2} * L_{element}$$

Where $F_{axial@key\ point}$ (lbs) is the axial force at a key point along the member, $F_{previous\ key\ point}$ (lbs) is the axial force at the previous key point (were walking along the axial force graph 'left' to 'right'), $v_{top\ left}$ (plf) is the shear force at the top left edge of the chord span (*not the element!*) within the span of the given element, $v_{bottom\ right}$(plf) is the shear at the bottom right of the chord span within the span of the given element, and $L_{element}$ is the length of the chord across the given element section. For the purposes of using this equation and understanding signs just pick a left and a right (even if that actually means bottom to top for example) in order to walk through the graph. There will often be lots of zeros for values that are plugged in since at the edges there is no above or below shear, and the axial forces should start and end at zero. These allow you to make useful simplifications depending on the case, but for notches there are many cases so always keep in mind you can just walk through the whole equation.

Keep in mind that this equation solves for the axial force at key points. If you take this equation one time across an entire diaphragm, you should get zero. If you want to get the maximum force in a diaphragm using only one span you need to find a key point in the middle where the chord force will be at its greatest, or use the simple diaphragm equation from 2.1 over the depth (including the notch, make it shorter to be more conservative) to get an estimate.

5 Analysis of Diaphragms With Holes

Holes are a very complex case since you have to break out so many sub diaphragms and transfer diaphragms that thinking through and accounting for all of the statics values becomes difficult.

5.1 Solve Shears for Sub-diaphragms (Accounting for Holes)

Draw Sub-diaphragms

The sub diaphragms consist of transfer diaphragms and any other subsections of the main diaphragm you need to do your analysis. Importantly you want to break the area above and below a hole into two equal sub-diaphragms even though there are no collectors there. Since the moment is zero at midspan above and below the hole we can use this boundary condition to do the calculations. Transfer diaphragms would have already been marked out in Step 1.1, but to reiterate they are sections that act to transfer shear across their area into shear walls. When you have a collector that accumulates axial forces but is not along a shear wall, like at the edges of holes, the diaphragm itself transfers the axial forces to a parallel shear wall using the shear capacity of the transfer diaphragm. This shear loading needs to be calculated and designed for.

NEED A DIAGRAM HERE

Some of this work has already been done when we created the original shear graph. The key difference is that holes concentrate the shear forces into the transfer diaphragms. These regions would take more shear than originally calculated off the basic shear diagram since that was done assuming the holes didn't exist. Another big difference has to do with holes, notably that the distributed load, w, is no longer entirely accurate. For seismic loads the load should be distributed to the top of the hole and the bottom of the hole relative to their depths, as the seismic load is more accurately a force per area (psf). For wind there are different windward and leeward loads, which in normal diaphragms can just be combined, but for holes need to be applied to the top or bottom as appropriate, remembering the load is reversible. What this means is that, unless otherwise mentioned, *for all sub-diaphragm calculations moving forward* you need to use the broken up distributed load on the top of the hole and the bottom of the hole.

Find Midspan Boundary Conditions

For a hole, the first step would be to calculate the chord forces at midspan of the topmost and bottommost chords of the opening. This can be done with the following equation:

$$F_{mid} = \left(V * x - \frac{w * x^2}{2}\right) / d$$

Where F_{mid} (lbs) is the chord force at midspan of the opening at the topmost and bottommost chords, V (lbs) is the relevant side of the shear wall reaction found at one of the ends of the analysis diaphragm, x (ft) is the distance from the shear wall to the point midspan of the opening, w (plf) is the applied load (ignoring the hole), and d (ft) is the depth of the entire diaphragm (ignoring the hole) from topmost to bottommost chords. The chords at midspan not at the very top and very bottom should have *zero* chord force at that point as they act as the point of inflection of the smaller hole beams and the center depth of the larger diaphragm beam.

Resolve Shears Across Sub-diaphragms

The next step is to resolve the shears across the four sub-diaphragms free body diagrams above and below the hole using our understanding of shear diagrams. Work consistently in one direction. Pull the shear (lbs) off of the basic shear diagram at the point where the hole opens up. Distribute the shear to the top and bottom of the hole with:

$$V_i = \frac{d_i * V_{basic}}{\Sigma d}$$

Where V_i (lbs) is the shear for either the top or bottom sub-diaphragm on the side close to the transfer diaphragm, V_{basic} (lbs) is the basic shear diagram shear, d_i (ft) is the depth of either the top or bottom sub-diaphragm, and Σd (ft) is the sum of the top and bottom sub-diaphragm depths (equivalent to the total depth minus the hole depth). There should be compression on the topmost chord and tension on the bottommost chord, assuming the analysis diaphragm has a 'downward' applied load.

We can find the vertical shear between the two hole elements with the linear slope equation,

$$V_{end} = V_{start} + w * x$$

walking along the shear graph with V_{start} (lbs) being the shear found at the hole side of the transfer diaphragm on the basic shear diagram. Note that now w is different, creating a deviation from the basic shear graph, and resulting in a different value for the shear midspan at the top elements versus the bottom elements. The sum of these two shears should result in the shear calculated off of the basic shear diagram at the same point. Now again walk along the shear graph from midspan to the other edge of the hole, again noting the deviations caused by the different w's. With the relevant shears we can convert them to shears per unit length along each edge of the sub diaphragms with the earlier equation, $v = \frac{V}{d}$.

Find the Chord Forces

Now we only have two unknown chord forces on each free body diagram, so we can use basic statics principles to find the remaining unknowns. First we take the sum of moments about a point that passes through one of the unknown forces, keeping in mind the midspan force, applied shear not passing through the point, and w. Solve for the other unknown chord force, then take the sum of forces in the chord direction to get the final unknown force. Repeating this for all 4 sub-diaphragms and the first part of the analysis diaphragm is complete.

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5.2 Solve Transfer Diaphragms (Using Sub-diaphragms)

This is a complicated section, so first I will work through a general derivation, then give more specific equations for hole problems.

The forces we have on the transfer diaphragms are a complex mix of vertical and horizontal forces applied over varying sections of the element that are difficult to consolidate into individual element shear forces for design. We already know the simple vertical shear forces from the transfer diaphragm since it is a part of the analysis diaphragm, that's the basic shear diagram, but we don't know how the shears are transmitted horizontally from the elements above and below the hole into the transfer diaphragm. Just looking at the *horizontal* point loads applied by the chords we see that they are in horizontal force equilibrium but not in moment equilibrium. The arrangement of the *vertical* forces creates the equal and opposite moment to compensate for the imbalance, meaning that solving the sum of moments about any point of the transfer diaphragm considering all the loads in both dimensions will come to zero. Since this free body diagram only works in two dimensions, but we want the forces in just the horizontal dimension, we need to find an equivalent static system for the vertical forces that generates the same moment with horizontal forces.

Sum M from Vertical F + Sum M from Horizontal F = 0
Sum M from Vertical F = Sum M from Horizontal F Couple
Sum M from Horizontal F Couple + Sum M from Horizontal F = 0
Sum M from Horizontal F = - Sum M from Horizontal F Couple
Sum M from Horizontal F Couple / Lever Distance = Horizontal F Couple

This horizontal force couple *is itself* the shear force transmitted from the parts above and below the hole into the transfer diaphragm. Interestingly this simplifies out to mean that we only need to know the horizontal forces and some distance measurements in order to find the shears despite the very real presence of other forces. Note that the horizontal force couple *includes* the applied forces at those points (the topmost and bottommost chord forces), they do not need to be double counted in the sum of moments from horizontal forces term. Note that the force at the top of the transfer diaphragm will likely not be the same as the force at the bottom of the diaphragm, which makes sense since these numbers are the sum of the force couple and existing forces, they will likely not add in a perfect way. With the force couple everything should still horizontally balance out. Also again we have the horizontal forces already solved, so we can start from that point and simplify the whole equation down into:

$$F_{couple} = \Sigma\left(F_{i,\,horizontal} * d_i\right) * \left(\frac{1}{d_{couple}}\right)$$

Where F_{couple} (lbs) is the shear force we are looking for, $F_{horizontal}$ (lbs) is a horizontal force applied at the side of the transfer diaphragm by an interior chord, d_i (ft) is the perpendicular distance from one edge of the transfer diaphragm to the relevant $F_{horizontal,}$ and d_{couple} (ft) is the depth of the whole transfer diaphragm, the perpendicular length of the force couple.

Find Applied Shears and Plot Them Against the Transfer Diaphragm

For a hole we can use more concrete terms to better define the equation so that it is easier to use. The transfer diaphragm is broken up by the chords into three elements. There are four horizontal chord forces coming from the hole side, the vertical shear forces applied by the parts above and below the hole, the vertical distributed load, and the vertical support shear applied at the opposite side of the transfer diaphragm. All of these forces should balance out to a sum of moments of zero. All we need for the equation are the two horizontal chord forces applied to the transfer diaphragm, everything else is accounted for in the earlier derivation.

NEED A DIAGRAM HERE given unsolved diaphragm

$$V_{top} = \left(- F_B * d_{bottom\ to\ B} - F_C * d_{bottom\ to\ C}\right) * \left(\frac{1}{d_{couple}}\right)$$

$$V_{bottom} = \left(F_B * d_{top\ to\ B} + F_C * d_{top\ to\ C}\right) * \left(\frac{1}{d_{couple}}\right)$$

Note the signs and distances, these are technically moments, for example the bottom section shear is found with a moment taken about the top edge of the transfer diaphragm. The forces assumed tension and the distances need to be the relevant lever arms. For the middle section we can use horizontal equilibrium across the bottom element:

$$V_{middle} = V_{bottom} - F_C$$

With the chord force acting like a discontinuity in the constant horizontal shear graph. The signs assume a positive shear in the bottom element and an applied force in tension to the right; these forces oppose each other at line C following standard internal sign conventions for shear and axial forces. The signs are important to note as they are relevant to the net calculations. The direction of the chord forces are the discontinuities in the graph, the section shears are the heights of the graph between the discontinuities. If you plot this the signs over each section should be apparent. Make sure you are walking in a consistent direction, like bottom to top.

Finally all of these shear forces can be converted to shears per unit length with $v = \frac{V}{d}$ where d is now the width of the transfer diaphragm between the support collector and the edge of the hole (these are horizontal shears so depth is the other dimension compared to when we last used it).

Find Net Transfer Diaphragm Shears in Each Section

Refer back to the basic shear diagram drawn in Step 2 and find the edges of the transfer diaphragm. Within the transfer diaphragm span, note down the shears per unit length at those two points. Now separately add these two shears to each of the top, middle, and bottom shears, resulting in six total shear values. These are the net shears for the top section at the left and right edges, the middle section at the left and right edges, and the bottom section at the left and right edges. These are the design shears. Note down the maximum of the two shears (plf) in each section as the shear the plywood in that area will need to be designed for in this loading case. The signs when doing the addition can be confusing. The basic diagram shear should be obvious, and if you plotted out a shear diagram for the applied chord loads then you may be good to go. Conceptually the hole should concentrate the shear load into the center section, and proportionally reduce the load in the top and bottom sections.

NEED A DIAGRAM HERE solved diaphragm with horizontal shears, basic diagram, and net shears

5.3 Solve and Plot Axial Collector Forces (Using Transfer-diaphragms/Sub-diaphragms)

The sum of the shear on each side of a collector line minus the shear wall resistance is the equation for the axial graph at any point. Note that the shear found in the transfer diagrams differs from the shear found on the basic shear diagram. The sum of the net shears on the transfer diagrams should equal the shears found in the basic shear diagram, but since we're determining the forces along the depth of the collectors we want to use the more accurate shear values found at all the points along the collector.

There are no shear walls on edge collectors so this process is fairly simple. We know that a simple sum of forces along the collector equals zero, keeping in mind which direction the internal sign convention for shear points the forces where the diaphragms meet the collector. We can accumulate the shears per unit length up to each of the key points:

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$$F_{2@D} = 0$$

$$F_{2@C} = (-v_{transfer\ bottom\ right} + v_{hole\ bottom\ left}) * L_{bottom}$$

$$F_{2@B} = F_{2C} + (-v_{transfer\ middle\ right}) * L_{middle}$$

$$F_{2@A} = (-v_{transfer\ top\ right} + v_{hole\ top\ left}) * L_{top} = 0$$

These points provide all the key points needed to plot the axial force in the given collector. Note that this assumes that positive tension is pulling away from point D, and that all v's are positive local shear coordinates. The shear graph, of course, must close to zero with some rounding error.

5.4 Solve and Plot Chord Forces (Using Sub-diaphragms)

Since we already have all the element shear forces, and shear forces act across a plane in both dimensions, we can already directly solve the chord forces. We can draw the axial force graph starting from zero and walking along the chord with the collectors bounding the elements as key points.

Moving along the chord we accumulate the average net shear applied to the chord across that element times the length of the chord across that element. For this we can use the general equation for finding axial forces due to adjacent shear forces:

$$F_{chord@collector} = F_{previous} + \frac{(v_{top\ left} - v_{bottom\ left}) + (v_{top\ right} - v_{bottom\ right})}{2} * L_{element}$$

Where $F_{chord@collector}$ (lbs) is the axial chord force at a key point, $F_{previous}$ (lbs) is the axial chord force at the previous key point (we're walking along the graph), $v_{top\ left}$ (plf) is the shear force at the top left edge of the chord (*not the element!*), and $L_{element}$ is the length of the chord across the given element section. This equation works for all chords, but it can be simplified depending on the nature of the chord. Topmost and bottommost chords will have lots of zeros in the equation since there are no top or bottom shear forces respectively, we also know that we always start and end at zero, but the opening chords will also have zero axial force at midspan of the opening (as already solved in Step 5.1). Note that the usage of 'top left' and 'bottom right' is potentially confusing, so refer to the diagram to ensure you are using the correct shear forces. Signs are important here, the equation assumes walking left to right along the chord with positive shear forces and positive tension.

With all the key points found connect the dots to create an axial force graph for each chord in this loading case. The maximum magnitude axial chord force is the design axial force for this chord in this load case.

6 Final Work

6.1 Solve axial shear wall collector forces

All axial forces (shear wall, collector, chord, and strut) and be saved for the end and designed from the ground up with nothing but the design shears. The shears allow you to walk along a member and accumulate the forces to generate the axial force graph with the shear forces as the slope. The reason why shear walls are explicitly held until the end is that they require the diaphragms on both sides of them to be solved, since the combination of those shears is what the wall actually experiences. With collectors, chords, and struts you can solve them completely with just the single analysis diaphragm, and furthermore there's often some simplified equations that can be directly applied to each case without having to go back to statics.

Shear walls are a little more complicated than collectors since they have parts that resist the forces and less straightforward key points. There aren't easy equations to solve them since the ends of each shear wall are also key points and they don't necessarily have to line up with chords. The first step is to figure out what the shear wall resistance is. The easiest way to obtain this is off of the basic shear diagram, the difference in shear across the support. This could result in some error though since across steps 3 and 4 rounding error likely has meant that the sum of all of the applied forces has drifted away from the original reaction force. For automated solutions the rounding error will likely be very minimal and the basic diagram is sufficient. In order to be more confident that the shear diagram will close to zero, start by working on the net shear diagram.

Draw a Net Shear Diagram

Really need a diagram here

Graphically draw your shear wall collector line with the distinction between the two laid out, add dimensions wherever a shear wall/collector transition occurs, or wherever a chord runs into the wall and changes the loading. Next to this draw the shear loading (plf) from the left side, and next to that draw the loading from the right side (of course line everything up with your shear wall and dimensioned drawings). Steps 2-4 are for solving one diaphragm with a hole in it, in order to know the shears on the diaphragm across the shear wall you need to solve that diaphragm separately up to this point and use its design shears, or the shear wall is the edge of the structure and the shears across the shear wall are zero. For transfer diaphragms this should be the net shear for the given section. Now find the total load (lbs) on the shear walls, $V_{total} = \Sigma(v_i * d_i)$, taking the sum of all of the shear loads (plf) multiplied by all of their respective depths (ft) (all of the necessary information should already be laid out graphically to help organize solving this). Now find the shear per unit length in the shear walls themselves, $v_{sw} = V_{total} / d_{sw}$, dividing the total shear (lbs) by the depth of just the shear walls (ft). This process should make logical sense as the shear walls need to take and distribute all of the force applied to them as they act as the reaction force. Draw the shear resistance (plf) next to the previous drawings. Next to all of this draw a net force diagram for the collector/shear wall:

$$v_{net} = v_{right} - v_{left} - \left|v_{sw}\right|$$

Where v_{net} (plf) is the net shear applied to the collector/shear wall. The signs are important here, really what we want is the sum of the shear loads left and right minus the shear resistance from the shear wall. The internal sign conventions of shear mean that while the loading may be in the same direction across the shear wall, the direction of the shear flips from the left side to the right side. This is why we subtract left despite seeking the sum of the loads. Once the net shear diagram is found the axial force diagram is next.

Draw an Axial Force Diagram

Next to the net shear diagram (plf) draw the axial force diagram (lbs). This is found by simply walking along the wall's depth using the net shear as the slope. Multiply the net shear by its relevant depth and put a point there, connect it to the start of the graph, and do that for every key point. The graph will increase along collectors and decrease along shear walls. The graph should close out to zero with room for rounding error. The maximum magnitude axial force over each collector section is the design axial force for that collector in this load case.

6.2 Tabulate results

You need to collect and display all the relevant results of this analysis on a lateral force resisting system floor plan. This floor plan should include the design shears on each section of diaphragm, the axial collector and strut forces, and the chord forces. Depending on the floor plan this could become very busy and hard to read, and there may be diaphragms that will have smaller forces by observation. For this reason it's a bit arbitrary which values you show, the key point being that you want to communicate the peak loads in each area that define how the plywood, chords, or struts/collectors need to be designed. It's also important to note that you should have one result for each of these design values despite having multiple load cases. For example with a transfer diaphragm next to a hole you may have six different design shears, you want to pick the largest design shear of all of those cases to communicate on the LFRS plan.

6.3 Check Capacities

List some key capacities
Nailing patterns/blocking and plywood shear capacities
Lumber axial forces